ture of N_2 and He where separation has taken place, the shock wave contains gradients in both concentrations and in γ so that the shock wave relations are difficult to apply.

In measuring the rotational temperatures through the shock wave, the author has also found non-Boltzmann distributions, as have all electron beam investigators. The temperatures presented are approximated by merging the rotational distribution functions. There is a considerable controversy over the analysis of rotational temperatures (see Ref. 6) about which the author has little to offer at the present time.

Finally, the statement by R. E. Center that Eq. (11) should be modified by a ratio of the appropriate lifetimes for the excited states is an oversight on his part since this equation contains the ratio of Einstein's transition probabilities of emission (A_{nm} of He to N_2) which are directly related to the lifetimes by

$$\frac{1}{\tau} = \sum_{m} A_{nm}$$

This relationship is discussed in Ref. 7.

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Comments on Schlieren Measurements of the Inviscid Hypersonic Wake of a Sphere

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Wilson[1] has found that a sphere at high Mach number produces a well-defined trail 8 diam in width for thousands of diameters behind the body in a schlieren photograph. This phenomenon appears to be independent of Reynolds number and body material and is called the "inviscid wake" by Wilson. He finds the "edge" of this wake by finding the location where the film contrast is greatest and identifies this location as the place where the flowfield "density gradient was greatest." If this statement is modified to read: "the radial location on the schlieren photo where the integrated density gradient along the light path was greatest," then it is interesting to see if this straightforward interpretation of the inviscid wake seen on the schlieren agrees quantitatively with the expected "inviscid wake width."

Received May 2, 1968; revision received May 24, 1968.

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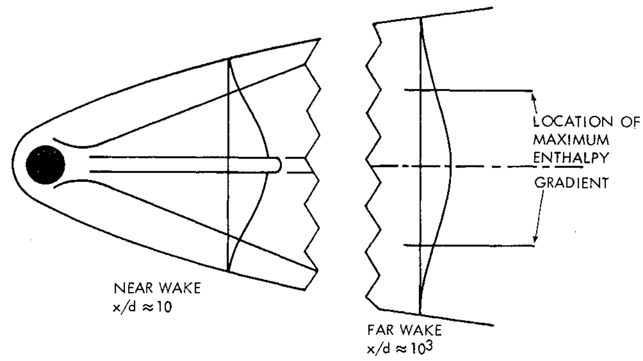

Fig. 1 Typical enthalpy profiles in near and far wake of a sphere.

Figure 1 shows typical enthalpy profiles expected in the near and far wake of a sphere for Reynolds numbers at which Wilson's data were taken. The inner "viscous wake" near the body is smeared out by x/d of order 10^3 but the shock-produced inviscid wake continues for several thousand diameters, its characteristic diffusion time being longer owing to its larger size.

Denoting the distance from the axis on a schlieren photograph by y, the film contrast in a schlieren system at y is proportional to the light deflection,[2] which is given by

$$\epsilon(y) = 2y \int_{y}^{\infty} \frac{(\partial n/\partial r)dr}{(r^2 - y^2)^{1/2}} \tag{1}$$

where r is the radial coordinate of the axisymmetric wake and n is the index of refraction of the wake. Since the wake electron density is much smaller than the neutral density, detectable variations in n are due only to neutral density variations and $n \sim \rho/\rho_\infty$.

We will attempt to evaluate (1) for a very simple wake model. The drag on the body is

$$D = 2\pi \int_{0}^{\infty} \rho u(u_\infty - u)rdr \tag{2}$$

at large x/d. Further at this distance $u \approx u_\infty$, the total enthalpy is approximately uniform, $H \approx H_\infty$, and so is the pressure $p \approx p_\infty$, so that $u(u_\infty - u) \approx h - h_\infty$ and $\rho/\rho_\infty \approx$

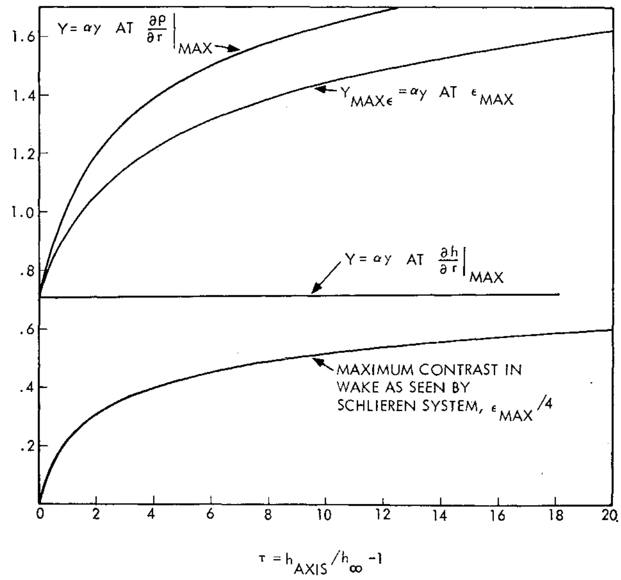

Fig. 2 Values of Y for maximum light deflection, enthalpy gradient, and density gradient. Also maximum schlieren light deflection $\epsilon/4$, vs $\tau = h_A/h_\infty - 1$. Gaussian enthalpy profile, $h/h_\infty = 1 + \tau \exp - Y^2$ is assumed.

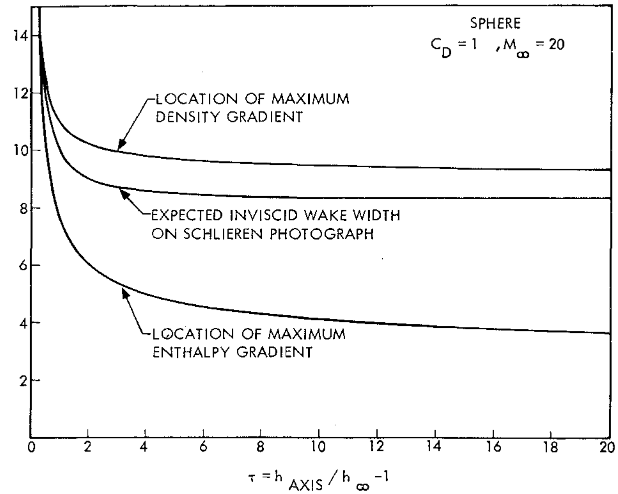

Fig. 3 Estimated radial location in body radii for maximum light deflection, enthalpy gradient, and density gradient in the inviscid wake of a sphere at $M_\infty = 20$ with $C_D = 1$.

h_∞/h; hence, (2) becomes

$$\frac{\gamma - 1}{4} M_\infty{}^2 C_D = \int_0^\infty \left(1 - \frac{h_\infty}{h}\right) r dr \tag{3}$$

wherein the body is taken to have unit radius. We may closely approximate† the enthalpy distribution in the far wake of a sphere by the Gaussian profile

$$(h - h_\infty)/(h_A - h_\infty) = \exp\{-\alpha^2 r^2\} \tag{4}$$

where h_A is the axis enthalpy and α measures the curvature of the profile at the axis. Inserting (4) in (3) and integrating gives a relation for α in terms of C_D, h_A/h_∞, M_∞,

$$\alpha = \left(\frac{\log h_A/h_\infty}{[(\gamma - 1)/2]M_\infty{}^2 C_D}\right)^{1/2} \tag{5}$$

Using (4), (1) becomes, with the substitution $\nu^2 = \alpha^2(r^2 - y^2)$,

$$\epsilon(y) = 4\tau Y e^{-Y^2} \int_0^\infty \frac{e^{-\nu^2}\, d\nu}{(1 + \tau e^{-Y^2} e^{-\nu^2})^2} \tag{6}$$

† See Fig. 1 of Ref. 3.

where we have denoted $\tau = h_A/h_\infty - 1$, $Y = \alpha y$, and we have taken $n = \rho/\rho_\infty$ since only relative values of ϵ are of interest. Thus, for small y, $\epsilon \sim y$ and, for large y, $\epsilon \to 0$ and, as expected, ϵ has a maximum at some value of y, not necessarily, however, where $\partial\rho/\partial r$ is a maximum as would be the case for a two-dimensional wake. The integral in (6) has been evaluated numerically and the variation of $\epsilon_{\max}/4$ and $Y_{\max\epsilon}$ with τ are shown in Fig. 2. Note that $\epsilon_{\max}$ is insensitive to τ for large τ and that $Y_{\max\epsilon} = (2)^{1/2}/2$ at $\tau = 0$. Also shown are the values of Y for maximum density gradient and enthalpy gradient. If we now specify C_D and M_∞, then (5) permits the expected inviscid wake width $y_{\max\epsilon} = Y_{\max\epsilon}/\alpha$ to be calculated as a function of τ. Figure 3 shows the result for $C_D = 1$, $M_\infty = 20$; the location of maximum enthalpy gradient $y_{\max\, \partial h/\partial r} = 2^{-1/2}\alpha^{-1}$ and maximum density gradient are also shown. We note that, for large τ, the expected inviscid wake radius is nearly double the radius of maximum enthalpy gradient. We further note that $y_{\max\epsilon} \approx 8$ (independent of τ for $\tau > 2$)‡ in remarkable agreement with Wilson's measurements. The origin of the inviscid wake phenomenon in Wilson's photos is therefore quantitatively explicable in a quite straightforward way.

References

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‡ The rapid variation of $Y_{\max\epsilon}$ with τ for $\tau < 2$ suggests the possibility of using the inviscid wake width as a thermometer for cases when $\tau < 2$; however, Fig. 2 indicates that the schlieren contrast may not be adequate for small τ. Another possibility is suggested by the fact that $y_{\max\epsilon}$ varies linearly with M_∞. Characteristics calculations indicate that $\tau > 5$ for $M_\infty > 10$; hence, the inviscid wake width for a sphere is a direct measure of M_∞ for $M_\infty > 10$. Wilson observes that the total variation of measured wake width is 20% for a velocity range of 19,000 to 23,000 fps, also a 20% variation. This spread is consistent with the preceding arguments, which suggest that the cause of this wake width variation is simply the variation in flight Mach number.